



Introduction to Vertical Coordinate Systems

Introduction to Vertical Coordinate (Reference) Systems

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Agenda

History

Why do we need a vertical reference?

Water flows down

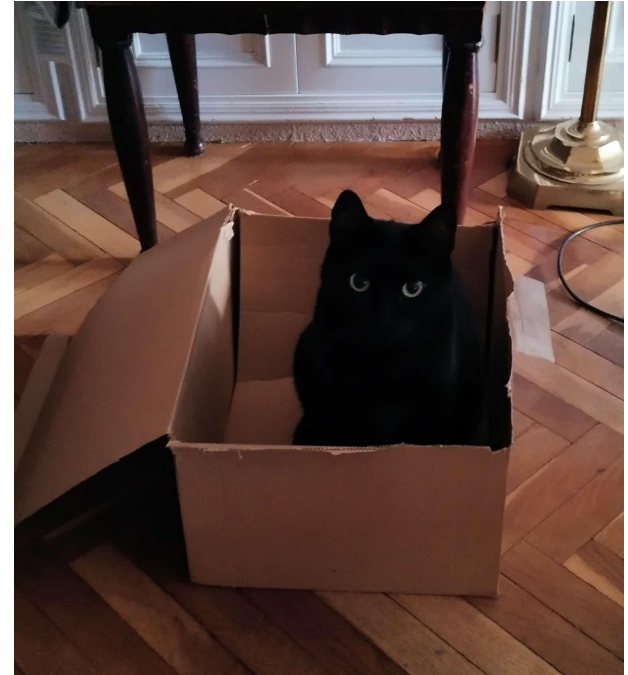
Equipotential surface

Mean Sea Level reference

Vertical Coordinate Reference System

GNSS

Geoid model









[Credit](#)

Aqua Claudia - Aqueduct

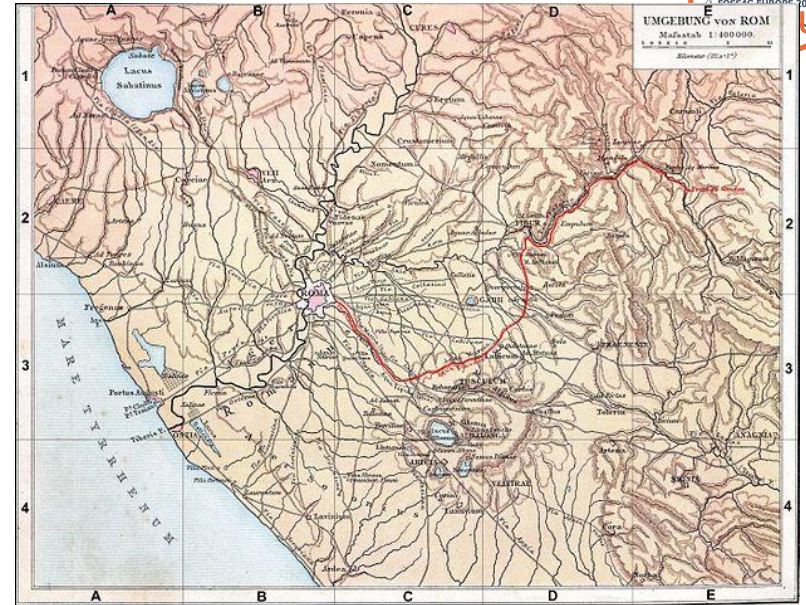
69 km

Finished 52 A.D.

Average slope 0.16%



[Credit](#)



[Credit](#)

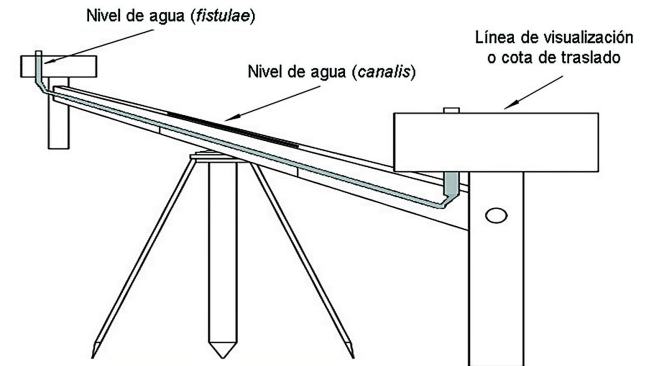
S·P·Q·R·



Chorobates

<https://en.wikipedia.org/wiki/Chorobates>

https://www.traianvs.net/pdfs/2004_roman_surveying.pdf

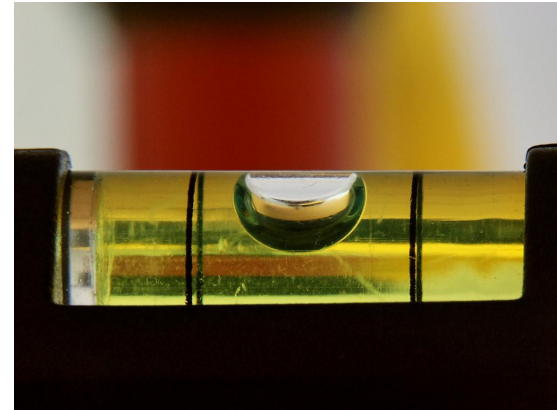


Why do we need a vertical reference?

... to measure **heights**.

... and slopes.

Water flows “**downwards**”. Pretty fast.



Up and down

Gravity defines Up and Down.



Gravity shapes the Earth.

Gravity rules.

Spoiler alert:

Gravity is not constant,
neither in magnitude nor direction.



<https://www.youtube.com/watch?v=AGsKPE3dk4k>

Flow down

The surface of a still liquid is an **“isopotential”** or **“equipotential”** surface:

- Points in this surface have the same gravitational potential.
- Moving in this surface does not need work.

Objects “fall” from a higher to a lower potential.

So water flows downwards.



The sea is flat



© [Wikipedia](#)

But we all know it is not flat

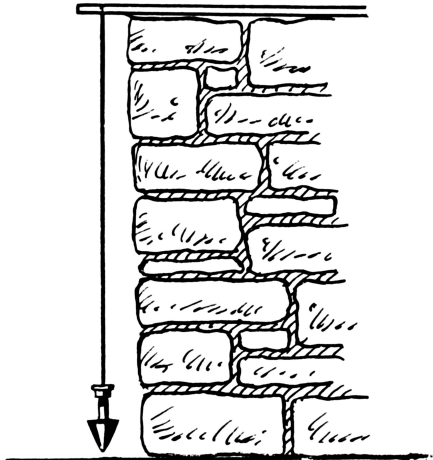


© [NOAA](#)

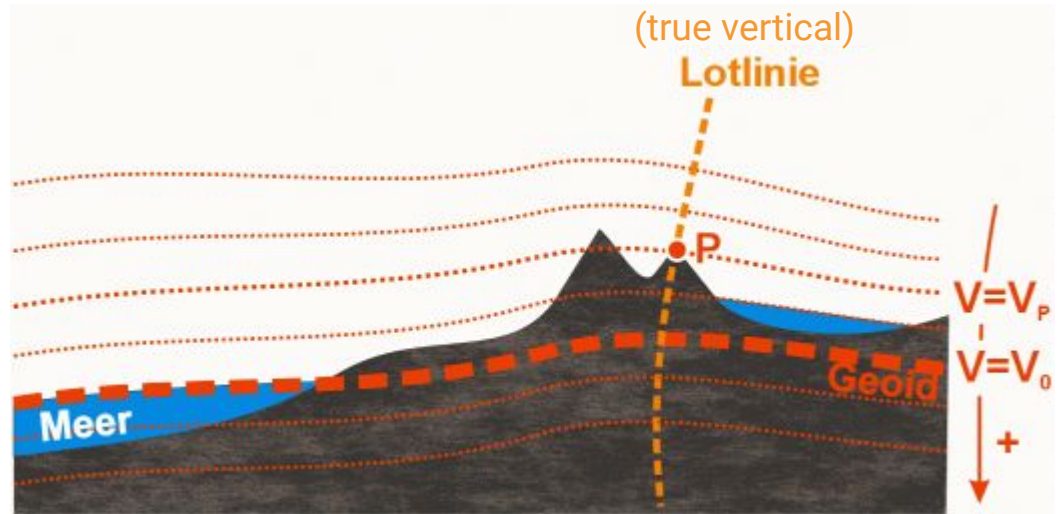
Equipotential surface \perp Plumbline

A plumb bob, or plummet, is a weight, usually with a pointed tip on the bottom, suspended from a string and used as a vertical reference line, or plumb-line.

“True vertical” is not a straight line! There we measure (orthometric) heights.



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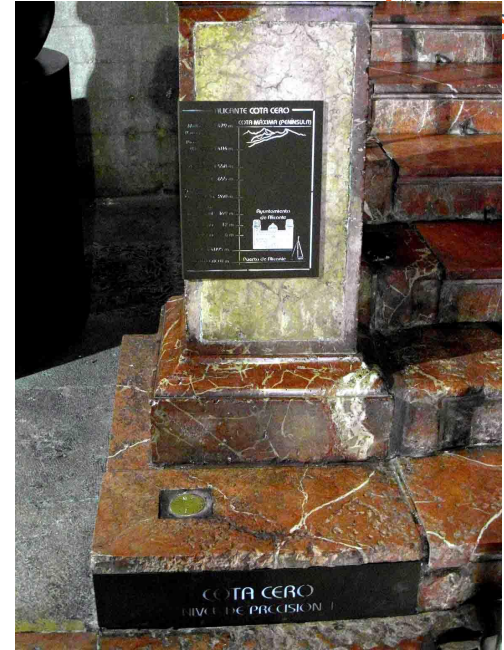
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Vertical reference - MSL

“Mean Sea Level” seems to be a good “0” (zero)

It is difficult to calculate that “mean”

Each country measured their own MSL, some in the XIX century, and still using that measurement.



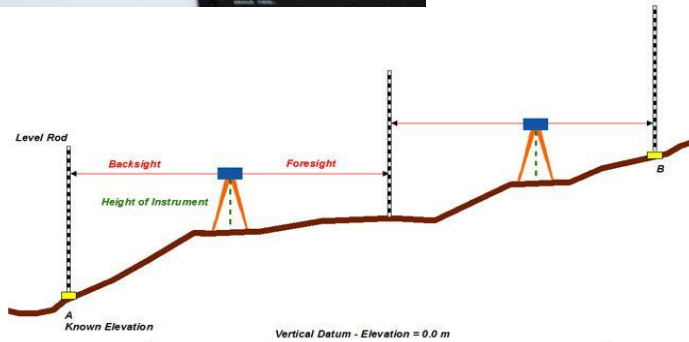
Alicante (Spain), between 1870 and 1872



Leveling inland

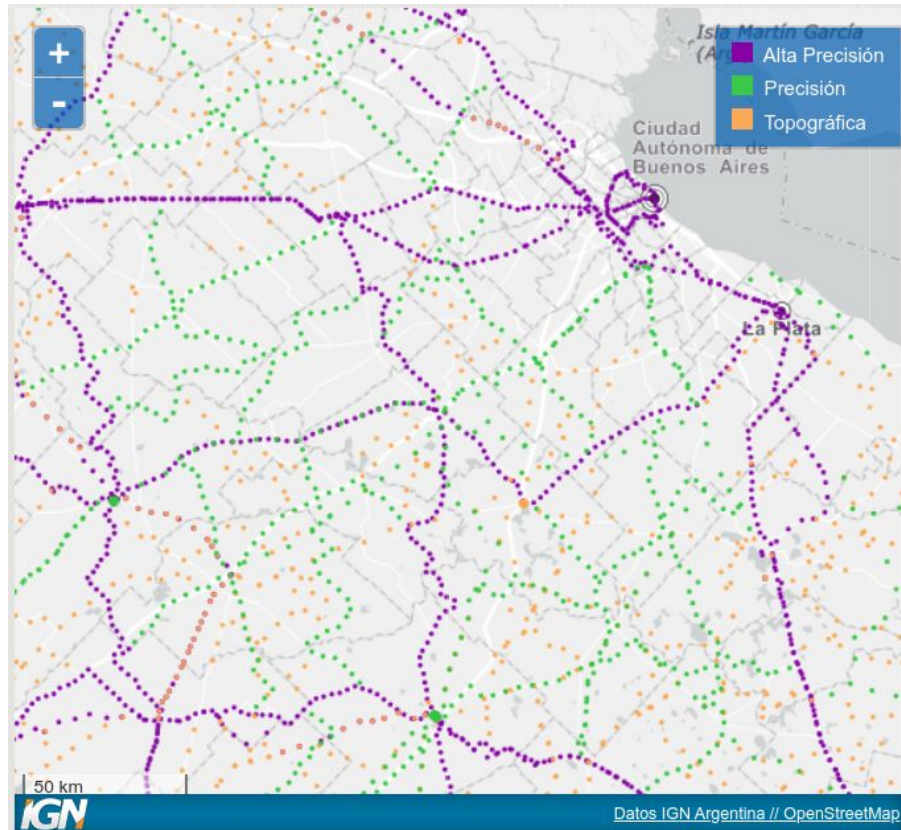


© [noaa](https://www.noaa.gov)



Leveling network

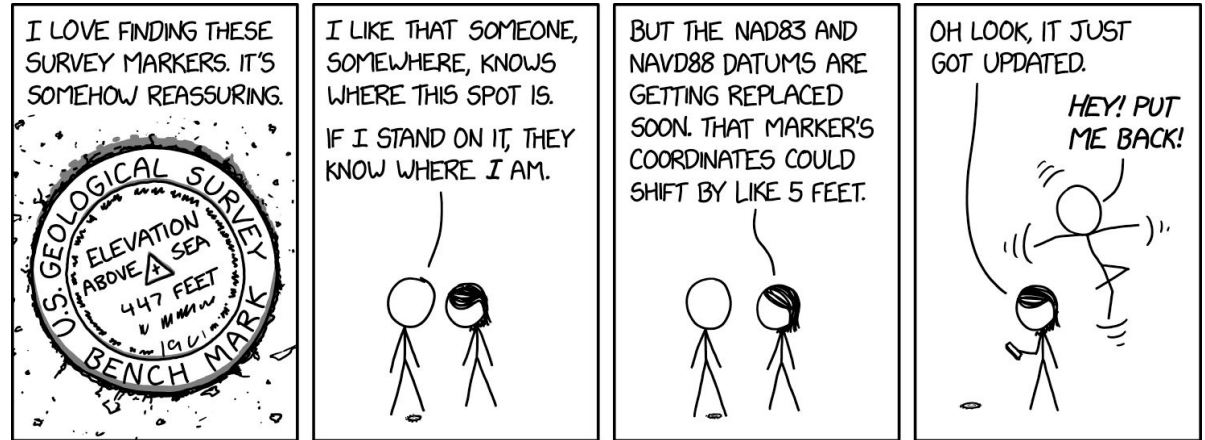
Detached from triangulation network (lat-lon)



Survey (elevation) marker



© [USGS](https://www.usgs.gov/)



© [xkcd](https://www.xkcd.com/)

Same point, different elevations

Each country defines their Vertical CRS.

Vaalsenberg

322.4 m for Germany and The Netherlands,
324.7 m for Belgium.



Vertical CRS in EPSG

Alicante height

Vertical CRS Details [VALID]	
NAME:	Alicante height
CODE:	5782
CRS TYPE:	Vertical

https://epsg.org/crs_5782/Alicante-height.html

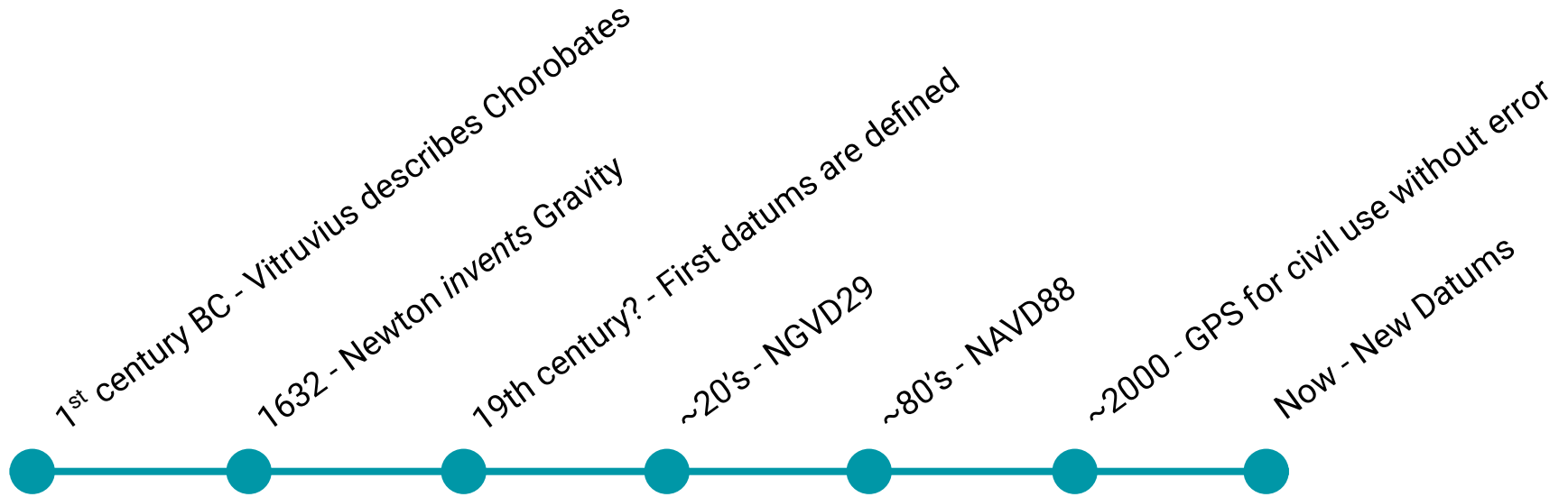
EPSG

EH2000 height

Vertical CRS Details [VALID]	
NAME:	EH2000 height
CODE:	9663
CRS TYPE:	Vertical

https://epsg.org/crs_9663/EH2000-height.html

History steps



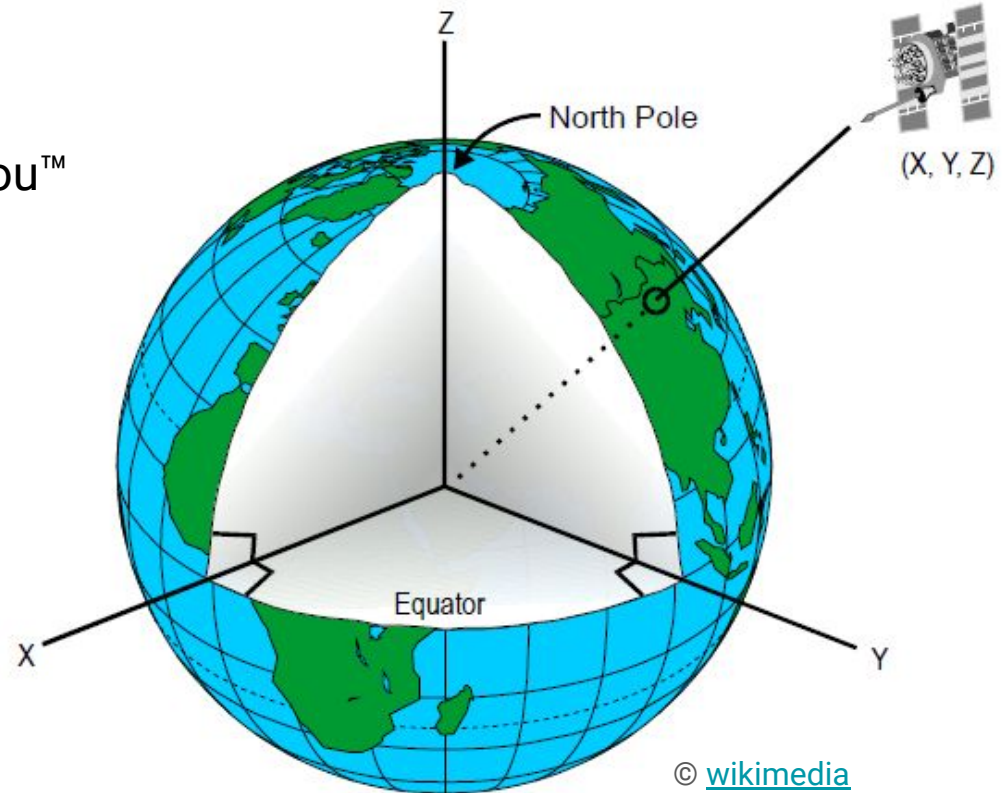
GNSS

Global Navigation Satellite System:

GPS™, Galileo™, Glonass™, Beidou™

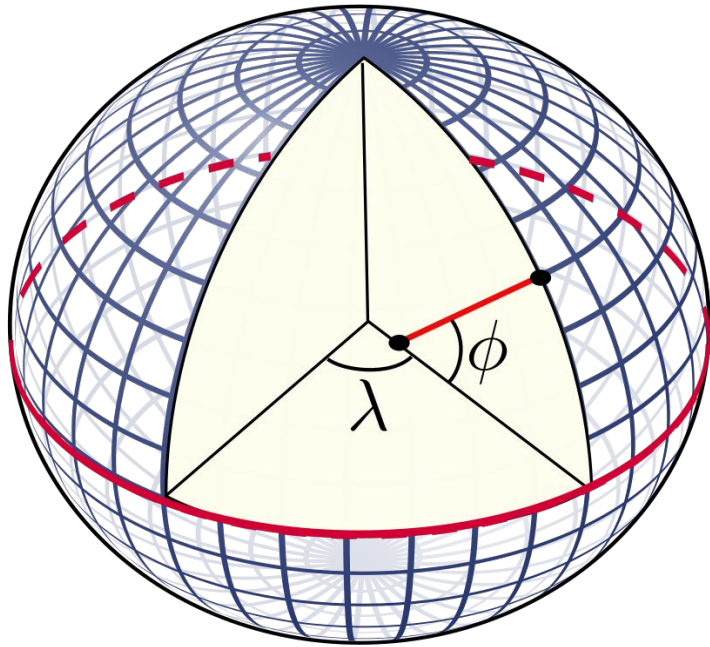


Credit: Elmar Brokmann



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Surveying with GNSS ... ellipsoid?



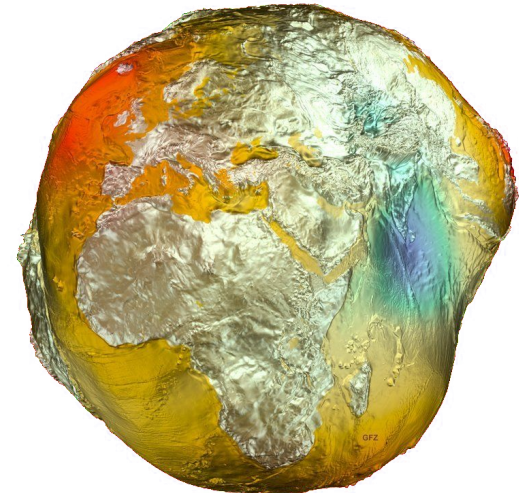
© [wikipedia](https://en.wikipedia.org/)



Credit: Elmar Brokmann

The Geoid

- Theoretical shape of the **gravitational equipotential surface**.
- It is the shape that the ocean surface would take under the influence of the gravity and rotation of Earth alone, if other influences such as winds and tides were absent.
- First defined by Gauss and Listing
- A smooth but **irregular surface** whose shape results from the **uneven distribution of mass** within and on the surface of Earth
- It is **NOT** the topographic surface of mountains and valleys.



Heights

- Gravity related -

Geopotential number and Dynamic height

Orthometric height

Normal height

<https://www.tandfonline.com/doi/pdf/10.1007/s11806-008-0074-z>

... and what about ellipsoidal heights?

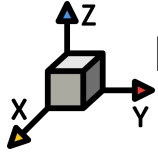




Satellites **XYZ** coordinates



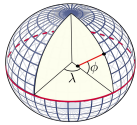
+ Time + Einstein + Black magic



Point in Geocentric coordinates (**x, y, z**)



+ "easy" math



Point in Geographic 3D (**lat, lon, h**) (ellipsoidal height)



+ Geoid model



Point in Compound CRS (**lat, lon, H**) (gravity related height)

21st
Century

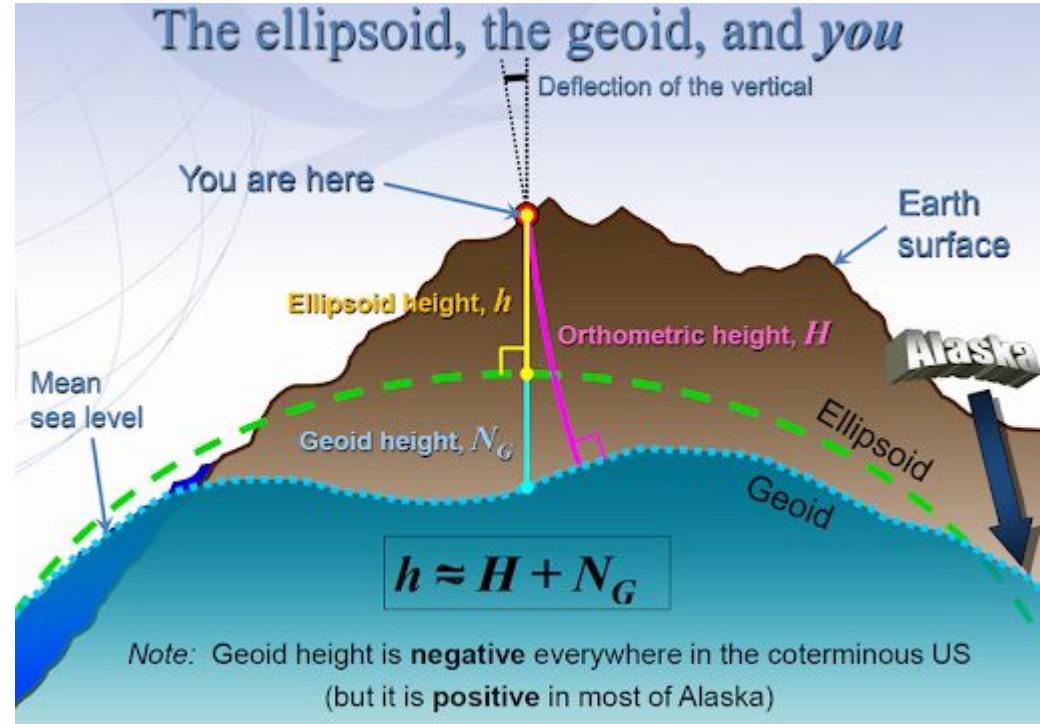
Ellipsoidal height - orthometric height

H : orthometric height

h : ellipsoidal height

N : Geoid undulation (or height)

$$H = h - N$$



Geoid model

Geoid model is usually stored as a **grid** (like a GeoTIFF).

That means an interpolation.

Referenced to a geographic CRS.

GNSS devices (including your phone) have a geoid model included to compute heights respect MSL, and not ellipsoidal heights.

The geoid model is just “an implementation detail”. What really defines the reference is a **Vertical Coordinate Reference System**.

Geoid model

EH2000 height (EE) - EST-GEOID2017

Alicante height (ES) - EGM08-REDNAP

NAVD88 height (US) - GEOID18 (and GEOID12B, GEOID09, ...)

ODN height (UK) - OSGM15

JGD2011 (vertical) height (JP) - GSIGEO2011

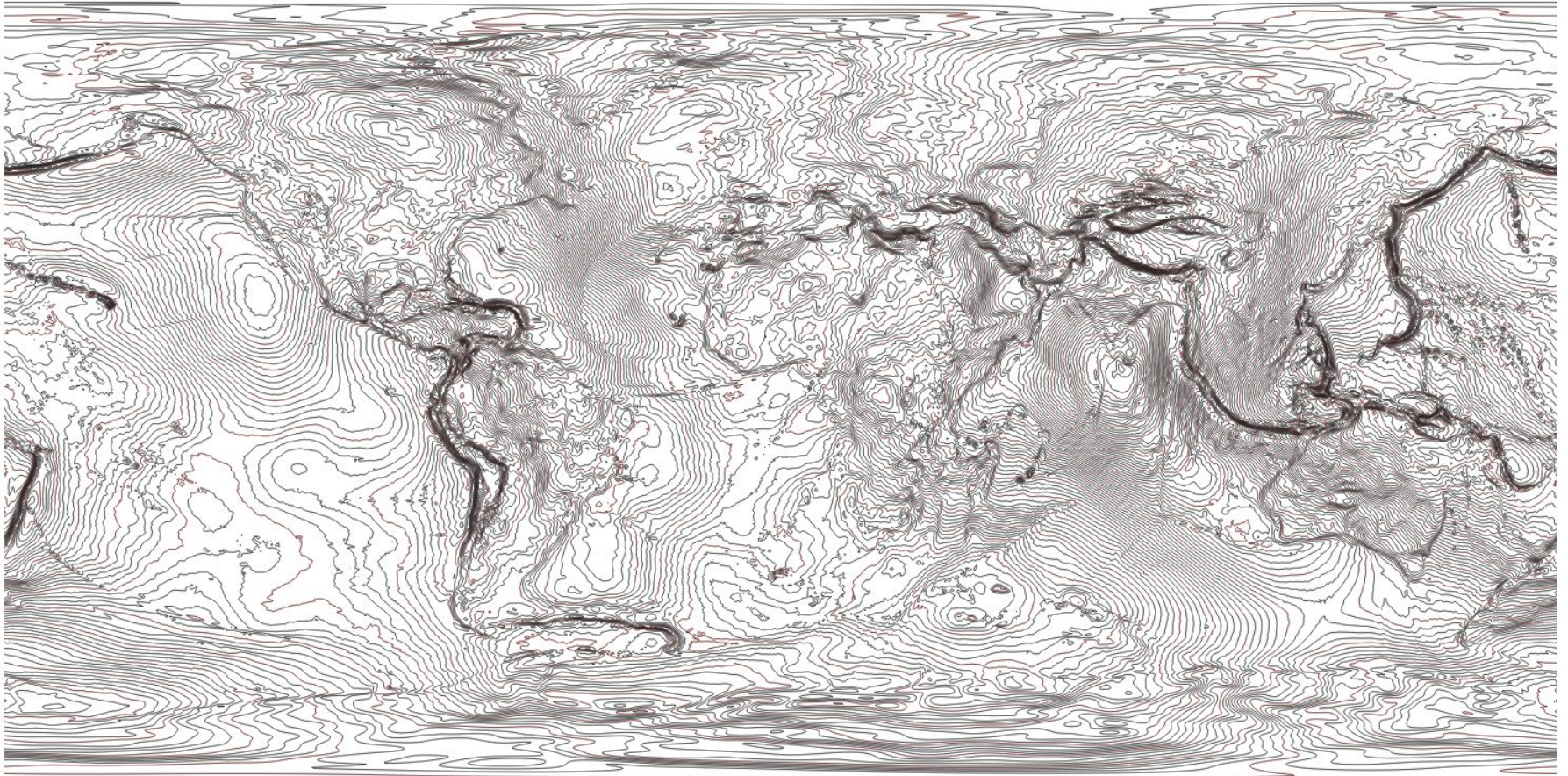


Warning

The image that you are going to see now is

NOT

describing altitudes, mountain elevations, or any
topographic accident.



EGM2008. Contour lines, 2m (range -100 to +80 m)

PROJ

Does PROJ understand Vertical CRSs?

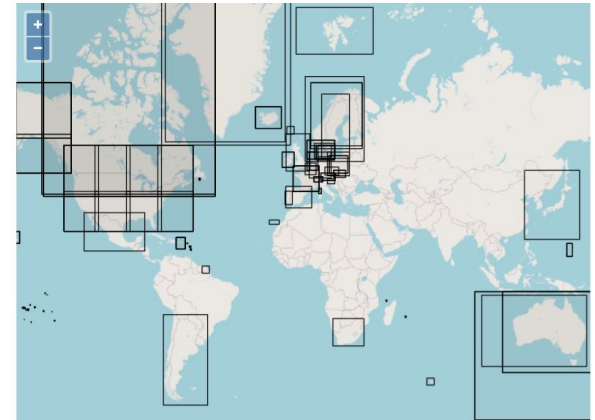
Yes, like in “cs2cs EPSG:4937 EPSG:25830+5782”

Does PROJ support geoid models?

Those included in cdn.proj.org

Types: Horizontal shift grids Geoid models Vertical shifts Velocity grids Deformations

Agencies: ar_ign at_bev au_ga au_icsm be_ign br_ibge ca_nrc ca_que_m
 de_lgvl_saarland dk_sdfe dk_sdfi es_cat_igcc es_ign eur_nkg fi_nls fr_ign
 pt_dgt se_lantmateriet si_gurs sk_gku uk_os us_nga us_noaa za_odngi



Agencies, please:

Add the **vertical coordinate reference system(s)** from your country to **EPSG** (or ISO).

- It makes **clear** to citizens and companies the VCRS to be used.
- Software will show **easily** that option.

Do you have different systems in some islands? Add them too!

Bonus points if you register the geoid model transformations ;)

And geoid models with [open license](#)!



Thanks for watching!

Javier Jimenez Shaw

<https://github.com/jjimenezshaw/>

